OFFICIAL CBSE 2020-21
SAMPLE QUESTION PAPER
MATHEMATICS (041) • CLASS XII

Time Allowed : 3 Hours | **Max. Marks : 80**

General Instructions :

1. This question paper contains two **Parts A and B**. Each part is compulsory. Part A carries **24** marks and Part B carries **56** marks.
2. **Part A** has Objective Type Questions and **Part B** has Descriptive Type Questions.
3. Both Part A and Part B have choices.

Part A :

1. It consists of two sections - **I and II**.
2. Section I comprises of 16 very short answer type questions.
3. Section II contains 2 case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt **any 4 out of 5 MCQs**.
4. Internal choice is provided in **5** questions of Section - I. Moreover internal choices have been given in both questions of Section - II as well.

Part B :

1. It consists of three sections - **III, IV and V**.
2. Section III comprises of 10 questions of **2 marks** each.
3. Section IV comprises of 7 questions of **3 marks** each.
4. Section V comprises of 3 questions of **5 marks** each.
5. Internal choice is provided in **3** questions of Section - III, **2** questions of Section - IV and **3** questions of Section - V. You have to attempt only one of the alternatives in all such questions.

PART - A

Section I

Questions in this section carry 1 mark each.

Q01. Check whether the function $f: R \to R$ defined as $f(x) = x^3$ is one-one or not.

OR

How many reflexive relations are possible in a set A whose $n(A) = 3$?

Q02. A relation R in $S = \{1,2,3\}$ is defined as $R = \{(1,1),(1,2),(2,2),(3,3)\}$. Which element (s) of relation R be removed to make R an equivalence relation?

Q03. A relation R in the set of real numbers **R** defined as $R = \{(a,b): \sqrt{a} = b\}$ is a function or not. Justify.

OR

An equivalence relation R in A divides it into equivalence classes A_1, A_2, A_3. What is the value of $A_1 \cup A_2 \cup A_3$ and $A_1 \cap A_2 \cap A_3$?

Q04. If A and B are matrices of order $3\times n$ and $m\times 5$ respectively, then find the order of matrix $5A-3B$, given that it is defined.

Q05. Find the value of A^2, where A is a 2×2 matrix whose elements are given by

$$a_{ij}=\begin{cases}1 & \text{if} \quad i\neq j\\ 0 & \text{if} \quad i=j\end{cases}.$$

OR

Given that A is a square matrix of order 3×3 and $|A|=-4$. Find $|\text{adj.} A|$.

Q06. Let $A=[a_{ij}]$ be a square matrix of order 3×3 and $|A|=-7$. Find the value of

$$a_{11}A_{21}+a_{12}A_{22}+a_{13}A_{23}$$

where A_{ij} is the cofactor of element a_{ij}.

Q07. Find $\int e^x(1-\cot x+\text{cosec}^2 x)\,dx$.

OR

Evaluate $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^2 \sin x \; dx$.

Q08. Find the area bounded by $y=x^2$, the x-axis and the lines $x=-1$ and $x=1$.

Q09. How many arbitrary constants are there in the particular solution of the differential equation $\frac{dy}{dx}=-4xy^2;\ y(0)=1$?

OR

For what value of n is the following a homogeneous differential equation :

$$\frac{dy}{dx}=\frac{x^3-y^n}{x^2y+xy^2}?$$

Q10. Find a unit vector in the direction opposite to $-\frac{3}{4}\hat{j}$.

Q11. Find the area of the triangle whose two sides are represented by the vectors $2\hat{i}$ and $-3\hat{j}$.

Q12. Find the angle between the unit vectors $\hat{a}$ and $\hat{b}$, given that $|\hat{a}+\hat{b}|=1$.

Q13. Find the direction cosines of the normal to YZ-plane?

Q14. Find the coordinates of the point where the line $\frac{x+3}{3}=\frac{y-1}{-1}=\frac{z-5}{-5}$ cuts the XY-plane.

Q15. The probability of A and B solving a problem independently are $\frac{1}{3}$ and $\frac{1}{4}$ respectively. If both of them try to solve the problem independently, what is the probability that the problem is solved?

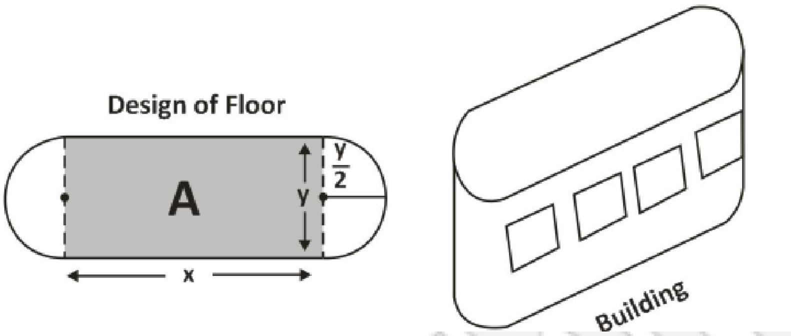
Q16. The probability that it will rain on any particular day is 50%. Find the probability that it rains only on first 4 days of the week.

Section II

Questions in this section carry 1 mark each.

Both the Case study based questions are compulsory. **Attempt any 4 sub-parts from each question 17 (i-v) and 18 (i-v)**.

Q17. An architect designs a building for a multi-national company. The floor consists of a rectangular region with semicircular ends having a perimeter of 200 m as shown below :



Based on the above information answer the following :

(i) If x and y represents the length and breadth of the rectangular region, then the relation between the variables is

(a) $x+\pi y=100$

(b) $2x+\pi y=200$

(c) $\pi x+y=50$

(d) $x+y=100$

(ii) The area of the rectangular region A expressed as a function of x is

(a) $\frac{2}{\pi}(100x-x^2)$

(b) $\frac{1}{\pi}(100x-x^2)$

(c) $\frac{x}{\pi}(100-x)$

(d) $\pi y^2+\frac{2}{\pi}(100x-x^2)$

(iii) The maximum value of area A is

(a) $\frac{\pi}{3200}\ m^2$

(b) $\frac{3200}{\pi}\ m^2$

(c) $\frac{5000}{\pi}\ m^2$

(d) $\frac{1000}{\pi}\ m^2$

(iv) The CEO of the multi-national company is interested in maximizing the area of the whole floor including the semi-circular ends. For this to happen, the value of x should be

(a) 0 m
(b) 30 m
(c) 50 m
(d) 80 m

(v) The extra area generated if the area of the whole floor is maximized is

(a) $\frac{3000}{\pi}$ m^2
(b) $\frac{5000}{\pi}$ m^2
(c) $\frac{7000}{\pi}$ m^2
(d) No change, Both areas are equal

Q18. In an office three employees Vinay, Sonia and Iqbal process incoming copies of a certain form. Vinay process 50% of the forms. Sonia processes 20% and Iqbal the remaining 30% of the forms. Vinay has an error rate of 0.06, Sonia has an error rate of 0.04 and Iqbal has an error rate of 0.03.

Based on the above information answer the following :

(i) The conditional probability that an error is committed in processing given that Sonia processed the form is
(a) 0.0210
(b) 0.04
(c) 0.47
(d) 0.06

(ii) The probability that Sonia processed the form and committed an error is
(a) 0.005
(b) 0.006
(c) 0.008
(d) 0.68

(iii) The total probability of committing an error in processing the form is
(a) 0
(b) 0.047

(c) 0.234

(d) 1

(iv) The manager of the company wants to do a quality check. During inspection he selects a form at random from the days output of processed forms. If the form selected at random has an error, the probability that the form is **NOT** processed by Vinay is

(a) 1

(b) $\frac{30}{47}$

(c) $\frac{20}{47}$

(d) $\frac{17}{47}$

(v) Let A be the event of committing an error in processing the form and let E_1, E_2 and E_3 be the events that Vinay, Sonia and Iqbal processed the form. The value of $\sum_{i=1}^{3} P(E_i | A)$ is

(a) 0

(b) 0.03

(c) 0.06

(d) 1

PART - B

Section III

Questions in this section carry 2 marks each.

Q19. Express $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right), -\frac{3\pi}{2} < x < \frac{\pi}{2}$ in the simplest form.

Q20. If A is a square matrix of order 3 such that $A^2 = 2A$, then find the value of $|A|$.

OR

If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = O$. Hence find A^{-1}.

Q21. Find the value(s) of k so that the following function is continuous at $x = 0$:

$$f(x) = \begin{cases} \frac{1-\cos kx}{x \sin x} & \text{if} \quad x \neq 0 \\ \frac{1}{2} & \text{if} \quad x = 0 \end{cases}.$$

Q22. Find the equation of the normal to the curve $y = x + \frac{1}{x}, x > 0$ perpendicular to the line $3x - 4y = 7$.

Q23. Find $\int \frac{1}{\cos^2 x(1-\tan x)^2} dx$.

OR

Evaluate $\int_0^1 x(1-x)^n dx$.

Q24. Find the area of the region bounded by the parabola $y^2 = 8x$ and the line $x = 2$.

Q25. Solve the following differential equation : $\frac{dy}{dx} = x^3 \text{cosec } y$, given that $y(0) = 0$.

Q26. Find the area of the parallelogram whose one side and a diagonal are represented by coinitial vectors $\hat{i} - \hat{j} + \hat{k}$ and $4\hat{i} + 5\hat{k}$ respectively.

Q27. Find the vector equation of the plane that passes through the point $(1,0,0)$ and contains the line $\vec{r} = \lambda \hat{j}$.

Q28. A refrigerator box contains 2 milk chocolates and 4 dark chocolates. Two chocolates are drawn at random. Find the probability distribution of the number of milk chocolates. What is the most likely outcome?

OR

Given that E and F are events such that $P(E) = 0.8, P(F) = 0.7, P(E \cap F) = 0.6$. Find $P(\bar{E} | \bar{F})$.

Section IV

Questions in this section carry 3 marks each.

Q29. Check whether the relation R in the set Z of integers defined as $R = \{(a, b) : a + b \text{ is divisible by } 2\}$ is reflexive, symmetric or transitive. Write the equivalence class containing 0 i.e., $[0]$.

Q30. If $y = e^{x \sin^2 x} + (\sin x)^x$, find $\frac{dy}{dx}$.

Q31. Prove that the greatest integer function defined by $f(x) = [x],\ 0 < x < 2$ is not differentiable at $x = 1$.

OR

If $x = a \sec\theta,\ y = b \tan\theta$ then, find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{6}$.

Q32. Find the intervals in which the function f given by

$f(x) = \tan x - 4x,\ x \in \left(0, \frac{\pi}{2}\right)$ is

(a) strictly increasing

(b) strictly decreasing.

Q33. Find $\int \frac{x^2 + 1}{(x^2 + 2)(x^2 + 3)} dx$.

Q34. Find the area of the region bounded by the curves $x^2 + y^2 = 4,\ y = \sqrt{3}x$ and x-axis in the first quadrant.

OR

Find the area of the ellipse $x^2 + 9y^2 = 36$ using integration.

Q35. Find the general solution of the following differential equation : $x\,dy - (y + 2x^2)dx = 0$.

Section V

Questions in this section carry 5 marks each.

Q36. If $A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, find A^{-1}.

Hence, solve the system of equations :

$x-2y=10,\ 2x-y-z=8,\ -2y+z=7$.

OR

Evaluate the product AB, where

$$A=\begin{bmatrix}1 & -1 & 0\\ 2 & 3 & 4\\ 0 & 1 & 2\end{bmatrix} \text{ and } B=\begin{bmatrix}2 & 2 & -4\\ -4 & 2 & -4\\ 2 & -1 & 5\end{bmatrix}.$$

Hence solve the system of linear equations

$x-y=3,\ 2x+3y+4z=17,\ y+2z=7$.

Q37. Find the shortest distance between the lines

$\vec{r}=3\hat{i}+2\hat{j}-4\hat{k}+\lambda(\hat{i}+2\hat{j}+2\hat{k})$ and $\vec{r}=5\hat{i}-2\hat{j}+\mu(3\hat{i}+2\hat{j}+6\hat{k})$.

If the lines intersect, find their point of intersection.

OR

Find the foot of the perpendicular drawn from the point $(-1, 3, -6)$ to the plane $2x+y-2z+5=0$. Also find the equation and length of the perpendicular.

Q38. Solve the following linear programming problem (L.P.P.) graphically.

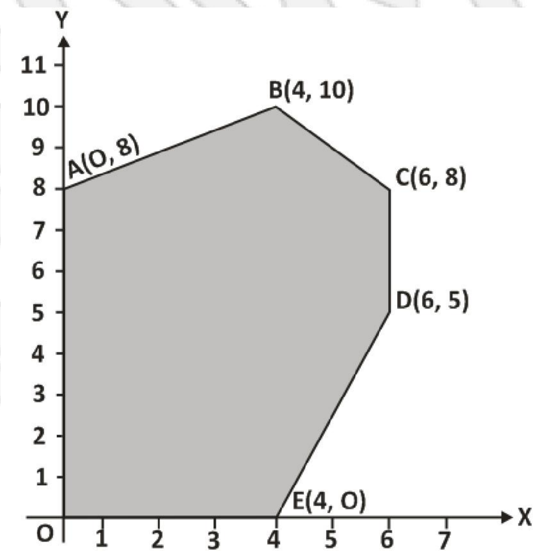
Maximize $Z=x+2y$.

Subject to constraints

$x+2y\geq 100,\ 2x-y\leq 0,\ 2x+y\leq 200;\ x, y\geq 0$.

OR

The corner points of the feasible region determined by the system of linear constraints are as shown below :



Answer each of the following :

(i) Let $Z=3x-4y$ be the objective function. Find the maximum and minimum value of Z and, also the corresponding points at which the maximum and minimum value occurs.

(ii) Let $Z=px+qy$, where $p, q>0$ be the objective function. Find the condition on p and q so that the maximum value of Z occurs at $B(4,10)$ and $C(6,8)$. Also mention the number of optimal solutions in this case.

PART - A

Section I

Q01. Let $f(x_1) = f(x_2)$ for some $x_1, x_2 \in R$

$\Rightarrow x_1^3 = x_2^3$

$\Rightarrow (x_1 - x_2)(x_1^2 + x_1x_2 + x_2^2) = 0$

$\Rightarrow (x_1 - x_2) = 0$ $\qquad \{\because (x_1^2 + x_1x_2 + x_2^2) \neq 0 \ \forall \ x_1, x_2 \in R$

$\Rightarrow x_1 = x_2$

Hence $f(x)$ is one-one.

OR

As the number of Reflexive relations defined on a set of n elements $= 2^{n(n-1)}$.

So, 2^6 reflexive relations are possible in the set A where $n(A) = 3$.

Q02. Note that $(1, 2) \in R$ but, $(2, 1) \notin R$.

So, R can't be symmetric.

Therefore, if (1, 2) is removed then only R will become an equivalence relation.

Q03. Since $\sqrt{a}$ is not defined for $a \in (-\infty, 0)$.

$\therefore$ $\sqrt{a} = b$ is not a function.

OR

$A_1 \cup A_2 \cup A_3 = A$ and $A_1 \cap A_2 \cap A_3 = \phi$.

Q04. In order to add matrices or to find their difference, the order of matrices must be **same**.

Here it is given that A and B are matrices of order $3 \times n$ and $m \times 5$ respectively.

So, values of m and n must be 3 and 5 respectively. Means the order of A and B must be 3×5.

Therefore, the order of $5A - 3B$ is 3×5.

Q05. Note that $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$\Rightarrow A^2 = A.A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$\therefore A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

That is, $A^2 = I_2$.

OR

Note that $|adj.\ A| = |A|^{n-1}$, where n is order of A.

So, we've $|adj.\ A| = (-4)^{3-1} = 16$.

Q06. $a_{11}A_{21} + a_{12}A_{22} + a_{13}A_{23} = 0$.

Q07. As $\int e^x \{f(x) + f'(x)\} dx = e^x f(x) + C$

So, $\int e^x(1-\cot x+\operatorname{cosec}^2 x)\,dx = e^x(1-\cot x)+C$.

Note that $f(x)=1-\cot x$ and $f'(x)=\operatorname{cosec}^2 x$.

OR

Let $f(x)=x^2\sin x$

$\Rightarrow f(-x)=(-x)^2\sin(-x)=-x^2\sin x=-f(x)$.

That means, $f(x)$ is an odd function.

$\therefore \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^2 \sin x\,dx = 0$.

Q08. Required area, $A=\int_{-1}^{1} x^2\,dx$

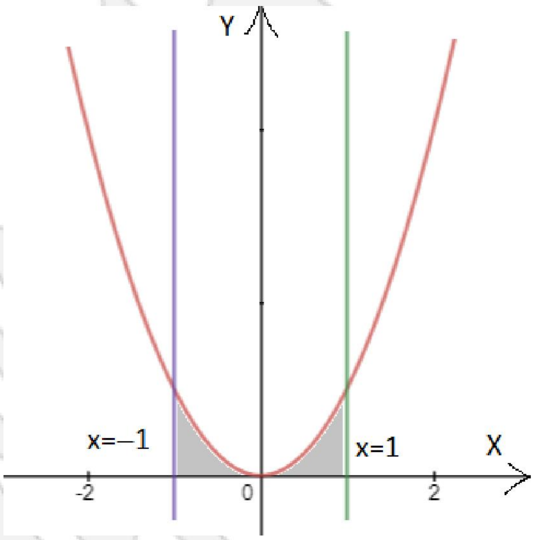
But $y=x^2$ is symmetrical about y-axis.

So, $A=2\int_0^1 x^2\,dx$

$\Rightarrow A=\frac{2}{3}[x^3]_0^1$

$\Rightarrow A=\frac{2}{3}[1^3-0^3]$

$\Rightarrow A=\frac{2}{3}$ Sq. units.



Q09. While solving a D.E. for the particular solution, we have to find the values of arbitrary constants. Therefore, there are no arbitrary constants left in the particular solution of the D.E. i.e., there are 0 arbitrary constants in the particular solution of the given differential equation.

OR

Value of n must be 3. Since the degree of x^3 is 3 and degree of y^n is n.

Q10. As the given vector $-\frac{3}{4}\hat{j}$ is along negative direction of y-axis so, the required unit vector in the direction opposite to $-\frac{3}{4}\hat{j}$ must be along the positive direction of y-axis.

Therefore, the required unit vector is $\hat{j}$.

Q11. Area of $\Delta=\frac{1}{2}\left|2\hat{i}\times(-3\hat{j})\right|=\frac{1}{2}\left|-6\hat{k}\right|=3$ Sq. units.

Q12. Consider $\left|\hat{a}+\hat{b}\right|^2=1$

$\Rightarrow |\hat{a}|^2+\left|\hat{b}\right|^2+2\hat{a}.\hat{b}=1$

$\Rightarrow 2\hat{a}.\hat{b}=1-1-1$

$\Rightarrow |\hat{a}|\left|\hat{b}\right|\cos\theta=-\frac{1}{2}$

$\Rightarrow 1\times1\times\cos\theta=-\dfrac{1}{2}$

$\Rightarrow \cos\theta=-\dfrac{1}{2}$

$\therefore\ \theta=\dfrac{2\pi}{3}$.

Q13. Equation of YZ-plane is $x=0$.

The d.c.'s of the normal to the YZ-plane : 1, 0, 0.

Q14. Let $\dfrac{x+3}{3}=\dfrac{y-1}{-1}=\dfrac{z-5}{-5}=\lambda$

The Coordinates of any random point on this line : $(3\lambda-3,-\lambda+1,-5\lambda+5)$.

Since the line cuts XY-plane so, we must have $(-5\lambda+5)=0$ $\Rightarrow \lambda=1$

Therefore, the required point is $(0, 0, 0)$.

Q15. P(problem is solved) $=1-$ P(problem isn't solved)

$$=1-\left(1-\frac{1}{3}\right)\times\left(1-\frac{1}{4}\right)$$

$$=1-\frac{2}{3}\times\frac{3}{4}$$

$$=\frac{1}{2}.$$

Q16. Required probability $=(50\%)(50\%)(50\%)(50\%)\times(100\%-50\%)(100\%-50\%)(100\%-50\%)$

$\Rightarrow \quad =\left(\dfrac{1}{2}\right)^4\left(\dfrac{1}{2}\right)^3$

$\Rightarrow \quad =\left(\dfrac{1}{2}\right)^7$.

Section II

Q17. **(i)** (b) Since the perimeter is 200 m so, $\left(\dfrac{1}{2}\times2\pi\times\dfrac{y}{2}\right)+x+x+\left(\dfrac{1}{2}\times2\pi\times\dfrac{y}{2}\right)=200$

$\Rightarrow 2\times\left(\dfrac{1}{2}\times2\pi\times\dfrac{y}{2}\right)+2x=200$

$\therefore\ \pi y+2x=200$.

(ii) (a) Area of the rectangular region, $A=xy$

$\Rightarrow A=x\left(\dfrac{200-2x}{\pi}\right)$

$\therefore A=\dfrac{2}{\pi}(100x-x^2)$.

(iii) (c) Note that $A=\dfrac{2}{\pi}(100x-x^2)$

$\Rightarrow \dfrac{dA}{dx}=\dfrac{2}{\pi}(100-2x)$

$\Rightarrow \dfrac{d^2A}{dx^2} = \dfrac{2}{\pi}(-2) < 0$ so, A is maximum.

For $\dfrac{dA}{dx} = 0,\ \dfrac{2}{\pi}(100 - 2x) = 0$

$\therefore x = 50$ m.

So, the maximum value of area $A = \dfrac{2}{\pi} \times 50 \times (100 - 50)$

$$\Rightarrow A = \frac{5000}{\pi}\ m^2.$$

(iv) (a) Area of the whole floor, $S = \dfrac{2}{\pi}(100x - x^2) + 2 \times \dfrac{1}{2}\pi \times \left(\dfrac{y}{2}\right)^2$

$\Rightarrow S = \dfrac{2}{\pi}(100x - x^2) + \dfrac{\pi}{4} \times \left(\dfrac{200 - 2x}{\pi}\right)^2$ $\left[\because y = \dfrac{200 - 2x}{\pi}\right.$

$\Rightarrow S = \dfrac{2}{\pi}(100x - x^2) + \dfrac{\pi}{4} \times \left(\dfrac{2(100 - x)}{\pi}\right)^2$

$\Rightarrow S = \dfrac{2}{\pi}(100x - x^2) + \dfrac{1}{\pi} \times (100 - x)^2$

Now $\dfrac{dS}{dx} = \dfrac{2}{\pi}(100 - 2x) - \dfrac{1}{\pi} \times 2(100 - x)$

Also $\dfrac{d^2S}{dx^2} = -\dfrac{4}{\pi} + \dfrac{1}{\pi} \times 2 = -\dfrac{2}{\pi} < 0$ means, S is maximum.

For $\dfrac{dS}{dx} = 0,\ \dfrac{2}{\pi}(100 - 2x) - \dfrac{1}{\pi} \times 2(100 - x) = 0$

$\Rightarrow \dfrac{2}{\pi}\left[(100 - 2x) - (100 - x)\right] = 0$

$\therefore x = 0$ m.

(v) (d) Note that the value of S at $x = 0$ m, $S = \dfrac{2}{\pi}(100 \times 0 - 0^2) + \dfrac{1}{\pi} \times (100 - 0)^2$

$\Rightarrow S = \dfrac{2}{\pi}(0) + \dfrac{1}{\pi} \times (100)^2$

That is, $S = \dfrac{10000}{\pi}\ m^2$

The extra area generated $= S - A = \dfrac{5000}{\pi}\ m^2$.

Therefore, the **extra area** is **same** as the **previous area** of rectangular region.
So, there is no change in both the areas.

Q18. **(i)** (b) Let A : event of committing an error in processing the form.
Also let E_1, E_2 and E_3 be the events that Vinay, Sonia and Iqbal processed the form, respectively.

Here $P(E_1) = 50\% = 0.5,\ P(E_2) = 20\% = 0.2,\ P(E_3) = 30\% = 0.3$.

Also $P(A \mid E_1) = 0.06,\ P(A \mid E_2) = 0.04,\ P(A \mid E_3) = 0.03$.

Now $P(A|E_2) = 0.04$.

(ii) (c) As $P(E_2 \cap A) = P(E_2) \times P(A|E_2)$

So, $P(E_2 \cap A) = (0.2)(0.04) = 0.008$.

(iii) (b) $P(A) = P(E_1)P(A|E_1) + P(E_2)P(A|E_2) + P(E_3)P(A|E_3)$

So, $P(A) = (0.5)(0.06) + (0.2)(0.04) + (0.3)(0.03)$

$\Rightarrow P(A) = 0.047$.

(iv) (d) Required probability $= 1 - P(E_1|A)$

$$\Rightarrow \quad = 1 - \frac{P(A|E_1)P(E_1)}{P(A|E_1)P(E_1) + P(A|E_2)P(E_2) + P(A|E_3)P(E_3)}$$

$$\Rightarrow \quad = 1 - \frac{(0.06)(0.5)}{0.047}$$

$$\Rightarrow \quad = 1 - \frac{0.030}{0.047}$$

$$\Rightarrow \quad = \frac{17}{47}.$$

(v) (d) $\sum_{i=1}^{3} P(E_i|A) = P(E_1|A) + P(E_2|A) + P(E_3|A)$

$$\Rightarrow \quad = \left(\begin{array}{c} \dfrac{P(A|E_1)P(E_1)}{P(A|E_1)P(E_1) + P(A|E_2)P(E_2) + P(A|E_3)P(E_3)} + \\ \dfrac{P(A|E_2)P(E_2)}{P(A|E_1)P(E_1) + P(A|E_2)P(E_2) + P(A|E_3)P(E_3)} + \\ \dfrac{P(A|E_3)P(E_3)}{P(A|E_1)P(E_1) + P(A|E_2)P(E_2) + P(A|E_3)P(E_3)} \end{array} \right)$$

$$\Rightarrow \quad = \frac{P(A|E_1)P(E_1) + P(A|E_2)P(E_2) + P(A|E_3)P(E_3)}{P(A|E_1)P(E_1) + P(A|E_2)P(E_2) + P(A|E_3)P(E_3)}$$

$$\Rightarrow \quad = 1.$$

PART - B
Section III

Q19. $$\tan^{-1}\left(\frac{\cos x}{1 - \sin x}\right) = \tan^{-1}\left[\frac{\sin\left(\frac{\pi}{2} - x\right)}{1 - \cos\left(\frac{\pi}{2} - x\right)}\right]$$

$$\Rightarrow \quad = \tan^{-1}\left[\frac{2\sin\left(\frac{\pi}{4} - \frac{x}{2}\right)\cos\left(\frac{\pi}{4} - \frac{x}{2}\right)}{2\sin^2\left(\frac{\pi}{4} - \frac{x}{2}\right)}\right]$$

$$\Rightarrow \quad = \tan^{-1}\left[\cot\left(\frac{\pi}{4} - \frac{x}{2}\right)\right]$$

$$\Rightarrow \quad = \tan^{-1}\left[\tan\left\{\frac{\pi}{2} - \left(\frac{\pi}{4} - \frac{x}{2}\right)\right\}\right]$$

$\Rightarrow \quad = \tan^{-1}\left[\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right]$ $\quad \left[\because -\frac{3\pi}{2} < x < \frac{\pi}{2} \Rightarrow -\frac{\pi}{2} < \left(\frac{\pi}{4}+\frac{x}{2}\right) < \frac{\pi}{2}\right.$

$\Rightarrow \quad = \frac{\pi}{4}+\frac{x}{2}.$

Q20. $A^2 = 2A$

$\Rightarrow |AA| = |2A|$

$\Rightarrow |A||A| = 8|A|$ $\quad \left[\because |AB| = |A||B| \text{ and, } |2A| = 2^3|A|\right.$

$\Rightarrow |A|\{|A|-8\} = 0$

Either $|A| = 0$ or, $|A|-8=0$

$\therefore |A| = 0$ or, 8.

OR

$A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$,

$5A = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}$ and, $7I = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

Now $A^2 - 5A + 7I = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$\Rightarrow A^2 - 5A + 7I = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\therefore A^2 - 5A + 7I = O$.

Pre-multiplying by A^{-1} both sides, we get

$A^{-1}(A^2 - 5A + 7I) = A^{-1}O$

$\Rightarrow A^{-1}AA - 5A^{-1}A + 7A^{-1}I = O$

$\Rightarrow A - 5I + 7A^{-1} = O$

$\Rightarrow 7A^{-1} = 5I - A$

$\Rightarrow 7A^{-1} = \left(\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\right)$

$\Rightarrow 7A^{-1} = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$

$\therefore A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$.

Q21. $\lim_{x\to 0}\frac{1-\cos kx}{x\sin x}$

$= \lim_{x\to 0}\frac{2\sin^2\left(\frac{kx}{2}\right)}{x\sin x}$

$$= \lim_{x\to 0}\frac{\dfrac{2\sin^2\left(\dfrac{kx}{2}\right)}{x^2}}{\dfrac{x\sin x}{x^2}}$$

$$= \frac{\lim\limits_{x\to 0}\dfrac{2\sin^2\left(\dfrac{kx}{2}\right)}{\left(\dfrac{kx}{2}\right)^2}\times\left(\dfrac{k}{2}\right)^2}{\lim\limits_{x\to 0}\dfrac{\sin x}{x}} \qquad \left[\because x\to 0 \therefore \frac{kx}{2}\to 0\right.$$

$$= \frac{2\times 1\times \dfrac{k^2}{4}}{1} = \frac{k^2}{2}$$

$\because f(x)$ is continuous at $x=0$

$\therefore \lim\limits_{x\to 0} f(x) = f(0)$

$\Rightarrow \dfrac{k^2}{2} = \dfrac{1}{2}$

$\Rightarrow k^2 = 1$

$\Rightarrow k = \pm 1$.

Q22. $y = x + \dfrac{1}{x}$

$\Rightarrow \dfrac{dy}{dx} = 1 - \dfrac{1}{x^2}$

Slope of normal $= \dfrac{x^2}{1-x^2}$

$\because$ normal is $\perp^{er}$ to $3x-4y=7$.

So, $\left(\dfrac{x^2}{1-x^2}\right)\left(\dfrac{3}{4}\right) = -1$

$\Rightarrow \dfrac{x^2}{1-x^2} = -\dfrac{4}{3}$

$\Rightarrow x^2 = 4$

$\Rightarrow x = 2 \qquad (\because x > 0)$

When $x=2$, $y = 2 + \dfrac{1}{2} = \dfrac{5}{2}$

$\therefore$ Equation of normal : $y - \dfrac{5}{2} = -\dfrac{4}{3}(x-2)$

$\Rightarrow 8x + 6y = 31$.

Q23. Let $I = \displaystyle\int \frac{1}{\cos^2 x(1-\tan x)^2}\,dx$

$\Rightarrow I = \int \frac{\sec^2 x}{(1-\tan x)^2} dx$

Put $1-\tan x = y$, so that $\sec^2 x\, dx = -dy$

Now, $I = \int \frac{-dy}{y^2}$

$\Rightarrow I = \frac{1}{y} + C$

$\Rightarrow I = \frac{1}{1-\tan x} + C$

OR

Let $I = \int_0^1 x(1-x)^n\, dx$

$\Rightarrow I = \int_0^1 (1-x)[1-(1-x)]^n\, dx$

$\Rightarrow I = \int_0^1 (1-x)x^n\, dx$

$\Rightarrow I = \int_0^1 (x^n - x^{n+1})dx$

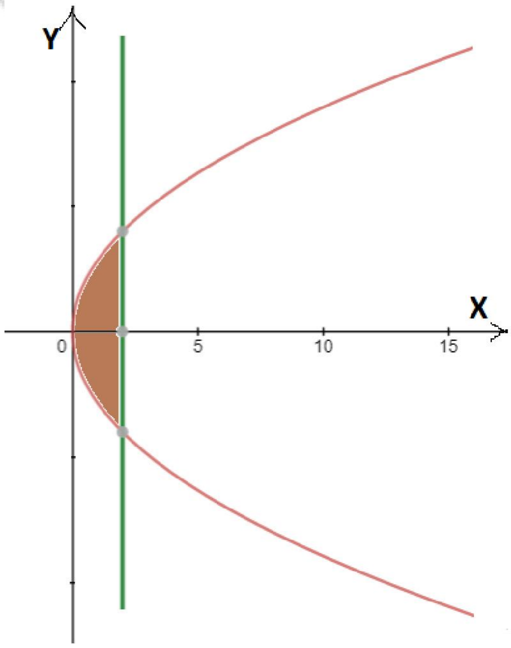
$\Rightarrow I = \left[\frac{x^{n+1}}{n+1} - \frac{x^{n+2}}{n+2}\right]_0^1$

$\Rightarrow I = \left(\frac{1}{n+1} - \frac{1}{n+2}\right) - 0$

$\Rightarrow I = \frac{(n+2)-(n+1)}{(n+1)(n+2)}$

$\therefore I = \frac{1}{(n+1)(n+2)}$.

Q24. The curve $y^2 = 8x$ is symmetrical about x-axis.



$\therefore$ Area $= 2\int_0^2 \sqrt{8x}\, dx$

$= 2\times 2\sqrt{2}\int_0^2 x^{\frac{1}{2}}\, dx$

$= 4\sqrt{2}\left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^2$

$= \frac{8}{3}\sqrt{2}\left[2^{\frac{3}{2}} - 0\right]$

$= \frac{8\sqrt{2}}{3} \times 2\sqrt{2}$

$= \frac{32}{3}$ Sq. units.

Q25. $\frac{dy}{dx} = x^3 \text{cosec } y$; $y(0) = 0$

$\Rightarrow \int \frac{dy}{\text{cosec } y} = \int x^3 dx$

$\Rightarrow \int \sin y\, dy = \int x^3 dx$

$\Rightarrow -\cos y = \frac{x^4}{4} + C$

$\because$ Given that $y = 0$, when $x = 0$

$\therefore -\cos 0 = \frac{0^4}{4} + C$

$\Rightarrow -1 = C$

Therefore, $\cos y = 1 - \frac{x^4}{4}$ is the required solution.

Q26. Let $\vec{a} = \hat{i} - \hat{j} + \hat{k}$ be the side and $\vec{d} = 4\hat{i} + 5\hat{k}$ be the diagonal of parallelogram.

$\because$ $\vec{a} + \vec{b} = \vec{d}$, where $\vec{b}$ is the other side of parallelogram.

$\therefore \vec{b} = \vec{d} - \vec{a} = 3\hat{i} + \hat{j} + 4\hat{k}$.

Now $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 3 & 1 & 4 \end{vmatrix}$

$\Rightarrow \vec{a} \times \vec{b} = -5\hat{i} - \hat{j} + 4\hat{k}$

So, the area of parallelogram $= |\vec{a} \times \vec{b}|$

$= \sqrt{25+1+16}$

$= \sqrt{42}$ Sq. units .

Q27. Let the d.r.'s of the normal to the plane be A, B, C.

Equation of the plane passing through the point $(1,0,0)$ is

$A(x-1) + B(y-0) + C(z-0) = 0$

$\Rightarrow A(x-1) + By + Cz = 0$ ...(i)

$\because$ plane (i) contains the line $\vec{r} = \lambda\hat{j}$

Rewriting the line in Cartesian form, we get : $\frac{x}{0} = \frac{y}{1} = \frac{z}{0} = \lambda$.

Note that the coordinates of random point on this line is $(0, \lambda, 0)$.

By (i), $-A + B\lambda = 0$ …(ii)

Also, $A.0 + B.1 + C.0 = 0 \Rightarrow B = 0$ (as the normal will be perpendicular to the line)

By (ii), $A = 0$.

Put values of A and B in (i), we get :

$0(x-1) + 0y + Cz = 0$ i.e., $z = 0$

That is, equation of the plane is : $\vec{r} \cdot \hat{k} = 0$ or, $(\vec{r} - \hat{i}) \cdot \hat{k} = 0$.

Q28. Let X denote the number of milk chocolates drawn.

So, values of X can be 0, 1, 2.

X	P(X)
0	$\frac{4}{6}\times\frac{3}{5}=\frac{12}{30}$
1	$\left(\frac{2}{6}\times\frac{4}{5}\right)\times 2=\frac{16}{30}$
2	$\frac{2}{6}\times\frac{1}{5}=\frac{2}{30}$

Clearly, most likely outcome is getting one chocolate of each type.

OR

$$P(\bar{E}\,|\,\bar{F})=\frac{P(\bar{E}\cap\bar{F})}{P(\bar{F})}=\frac{P(\overline{E\cup F})}{P(\bar{F})}=\frac{1-P(E\cup F)}{1-P(F)} \quad \ldots(i)$$

Now $P(E\cup F)=P(E)+P(F)-P(E\cap F)$

$=0.8+0.7-0.6=0.9$

Substituting value of $P(E\cup F)$ in (i)

So, $P(\bar{E}\,|\,\bar{F})=\frac{1-0.9}{1-0.7}$

$\Rightarrow P(\bar{E}\,|\,\bar{F})=\frac{0.1}{0.3}$

$\therefore\ P(\bar{E}\,|\,\bar{F})=\frac{1}{3}$.

Section IV

Q29. (i) Reflexive

Since, $a+a=2a$ which is even.

$\therefore\ (a,a)\in R\ \forall\ a\in Z$.

Hence, R is reflexive.

(ii) Symmetric

If $(a,b)\in R$, then $a+b=2\lambda \ \Rightarrow b+a=2\lambda$

$\Rightarrow (b,a)\in R$.

Hence, R is symmetric.

(iii) Transitive

If $(a,b)\in R$ and $(b,c)\in R$

then $a+b=2\lambda$ …(i)

and $b+c=2\mu$ …(ii)

Adding (i) and (ii) we get

$a+2b+c=2(\lambda+\mu)$

$\Rightarrow a+c=2(\lambda+\mu-b)$

$\Rightarrow a+c=2k$, where $\lambda+\mu-b=k$

$\Rightarrow (a,c)\in R$

Hence R is transitive.

Also for [0], let $(0, x) \in R$ for all $x \in Z$ i.e., $0 + x = x$ is divisible by 2.

That is, $x = 0, \pm 2, \pm 4, \pm 6, \ldots$.

Hence, $[0] = \{\ldots, -4, -2, 0, 2, 4, \ldots\}$.

Q30. Let $u = e^{x\sin^2 x}$ and $v = (\sin x)^x$ so that, $y = u + v$

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \qquad \ldots(i)$$

Now $u = e^{x\sin^2 x}$

Differentiating both sides w. r. t. x, we get

$$\Rightarrow \frac{du}{dx} = e^{x\sin^2 x}[x(\sin 2x) + \sin^2 x] \qquad \ldots(ii)$$

Also, $v = (\sin x)^x$

$\Rightarrow \log v = x \log(\sin x)$

Differentiating both sides w. r. t. x, we get

$$\frac{1}{v} \times \frac{dv}{dx} = x\cot x + \log(\sin x)$$

$$\Rightarrow \frac{dv}{dx} = (\sin x)^x [x\cot x + \log(\sin x)] \qquad \ldots(iii)$$

By (i), (ii) and (iii), we get

$$\frac{dy}{dx} = e^{x\sin^2 x}[x\sin 2x + \sin^2 x] + (\sin x)^x [x\cot x + \log(\sin x)].$$

Q31. $$\text{RHD} = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h\to 0} \frac{[1+h] - [1]}{h}$$

$$= \lim_{h\to 0} \frac{(1-1)}{h} = 0$$

$$\text{LHD} = \lim_{h\to 0} \frac{f(1-h) - f(1)}{-h}$$

$$= \lim_{h\to 0} \frac{[1-h] - [1]}{-h}$$

$$= \lim_{h\to 0} \frac{0-1}{-h}$$

$$= \lim_{h\to 0} \frac{1}{h}$$

$$= \frac{1}{0} = \infty.$$

Note that, LHD is not defined.

Therefore, $f(x)$ is not differentiable at $x = 1$.

OR

$y = b\tan\theta \Rightarrow \dfrac{dy}{d\theta} = b\sec^2\theta$ ...(i)

$x = a\sec\theta \Rightarrow \dfrac{dx}{d\theta} = a\sec\theta\tan\theta$ ...(ii)

So, $\dfrac{dy}{dx} = \dfrac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \dfrac{b\sec^2\theta}{a\sec\theta\tan\theta} = \dfrac{b}{a}\text{cosec}\,\theta$

Differentiating both sides w. r. t. x, we get

$$\frac{d^2y}{dx^2} = \frac{-b}{a}\text{cosec}\,\theta\cot\theta \times \frac{d\theta}{dx}$$

$$= \frac{-b}{a}\text{cosec}\,\theta\cot\theta \times \frac{1}{a\sec\theta\tan\theta} \quad \text{[using (ii)]}$$

$$= \frac{-b}{a.a}\cot^3\theta$$

$$\therefore \left.\frac{d^2y}{dx^2}\right]_{\text{at } \theta=\frac{\pi}{6}} = \frac{-b}{a^2}\left[\cot\frac{\pi}{6}\right]^3 = \frac{-b}{a^2}(\sqrt{3})^3 = -\frac{3\sqrt{3}\,b}{a^2}.$$

Q32. $f(x) = \tan x - 4x$

So, $f'(x) = \sec^2 x - 4$

(a) For $f(x)$ to be strictly increasing

$f'(x) > 0$

$\Rightarrow \sec^2 x - 4 > 0$

$\Rightarrow \sec^2 x > 4$

$\Rightarrow \dfrac{1}{\cos^2 x} > 4$

$\Rightarrow \cos^2 x < \dfrac{1}{4}$

$\Rightarrow \cos^2 x < \left(\dfrac{1}{2}\right)^2$

$\Rightarrow -\dfrac{1}{2} < \cos x < \dfrac{1}{2}$ $\left\{\because x \in \left(0, \dfrac{\pi}{2}\right)\right.$

$\Rightarrow \dfrac{\pi}{3} < x < \dfrac{\pi}{2}$

Hence, $f(x)$ is strictly increasing in $x \in \left(\dfrac{\pi}{3}, \dfrac{\pi}{2}\right)$.

(b) For $f(x)$ to be strictly decreasing

$f'(x) < 0$

$\Rightarrow \sec^2 x - 4 < 0$

$\Rightarrow \sec^2 x < 4$

$\Rightarrow \dfrac{1}{\cos^2 x} < 4$

$\Rightarrow \cos^2 x > \dfrac{1}{4}$

$\Rightarrow \cos^2 x > \left(\dfrac{1}{2}\right)^2$

$\Rightarrow \cos x > \dfrac{1}{2}$ $\left\{\because x \in \left(0, \dfrac{\pi}{2}\right)\right.$

$\Rightarrow 0 < x < \dfrac{\pi}{3}$

Hence, $f(x)$ is strictly decreasing in $x \in \left(0, \dfrac{\pi}{3}\right)$.

Q33. Put $x^2 = y$ to make partial fractions

$\dfrac{x^2+1}{(x^2+2)(x^2+3)} = \dfrac{y+1}{(y+2)(y+3)}$, where $y = x^2$

$\Rightarrow \dfrac{y+1}{(y+2)(y+3)} = \dfrac{A}{y+2} + \dfrac{B}{y+3}$

$\Rightarrow y+1 = A(y+3) + B(y+2)$ ...(i)

Comparing coefficients of y and constant terms on both sides of (i), we get

$A + B = 1$ and $3A + 2B = 1$

On solving these equations, we get :

$A = -1, B = 2$

Now, $\displaystyle\int \frac{x^2+1}{(x^2+2)(x^2+3)} dx = \int \left(\frac{-1}{x^2+2} + \frac{2}{x^2+3}\right) dx$

$\displaystyle = \int \frac{-1}{x^2+2} dx + 2\int \frac{1}{x^2+3} dx$

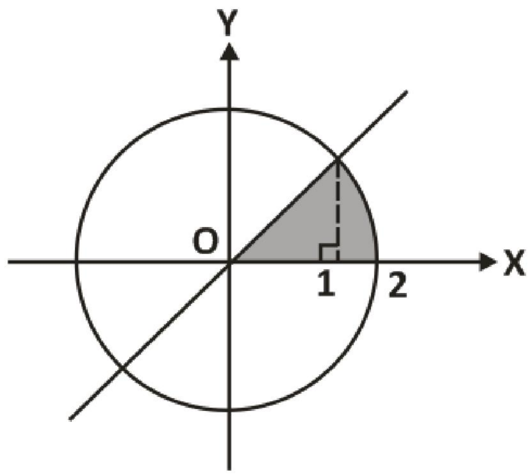
$\displaystyle = -\frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C$.

Q34. Solving $y = \sqrt{3}x$ and $x^2 + y^2 = 4$

We get $x^2 + 3x^2 = 4$

$\Rightarrow x^2 = 1$

$\Rightarrow x = 1$ (in first quadrant, $x > 0$)



Required Area $\displaystyle = \sqrt{3}\int_0^1 x\, dx + \int_1^2 \sqrt{2^2 - x^2}\, dx$

$\displaystyle = \frac{\sqrt{3}}{2}[x^2]_0^1 + \left[\frac{x}{2}\sqrt{2^2 - x^2} + 2\sin^{-1}\left(\frac{x}{2}\right)\right]_1^2$

$\displaystyle = \frac{\sqrt{3}}{2} + \left[2 \times \frac{\pi}{2} - \frac{\sqrt{3}}{2} - 2 \times \frac{\pi}{6}\right]$

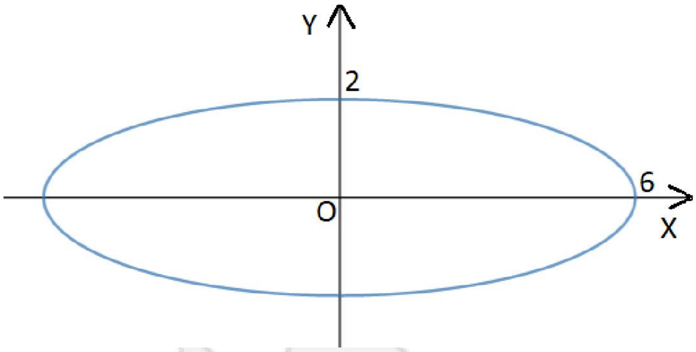
$=\frac{2\pi}{3}$ Sq. units.

OR

We have $x^2+9y^2=36$

That is, $\frac{x^2}{36}+\frac{y^2}{4}=1$

Note that the given ellipse is symmetrical about both the axes.



$\therefore$ Required Area $=4\times\frac{1}{3}\int_0^6\sqrt{6^2-x^2}\,dx$

$=\frac{4}{3}\left[\frac{x}{2}\sqrt{6^2-x^2}+18\sin^{-1}\left(\frac{x}{6}\right)\right]_0^6$

$=\frac{4}{3}\left[18\times\frac{\pi}{2}-0\right]$

$=12\pi$ Sq. units.

Q35. The given differential equation can be written as

$\frac{dy}{dx}=\frac{y+2x^2}{x}$

$\Rightarrow\frac{dy}{dx}-\frac{1}{x}y=2x$

Here $P=-\frac{1}{x}$, $Q=2x$

Now I.F. $=e^{\int P dx}=e^{-\int\frac{1}{x}dx}=e^{-\log x}=\frac{1}{x}$

The solution is,

$y\times\frac{1}{x}=\int\left(2x\times\frac{1}{x}\right)dx+C$

$\Rightarrow\frac{y}{x}=2x+C$

$\Rightarrow y=2x^2+Cx$.

Section V

Q36. We have $A=\begin{bmatrix}1 & 2 & 0\\ -2 & -1 & -2\\ 0 & -1 & 1\end{bmatrix}$

$\therefore\ |A|=1(-1-2)-2(-2-0)$

$\Rightarrow|A|=-3+4=1\neq0$.

Clearly A is nonsingular, therefore A^{-1} exists.

Consider A_{ij} be the cofactor of element a_{ij} of matrix A.

$A_{11} = -3,\ A_{12} = 2,\ A_{13} = 2$

$A_{21} = -2,\ A_{22} = 1,\ A_{23} = 1$

$A_{31} = -4,\ A_{32} = 2,\ A_{33} = 3$

$$\text{So, adj.A} = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{|A|}(\text{adj. A}) = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$$

Now the given equations are

$x - 2y = 10,\ 2x - y - z = 8,\ -2y + z = 7.$

The given equations can be written as

$$\begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

which is of the form $A^T X = B$.

$\Rightarrow X = (A^T)^{-1}B = (A^{-1})^T B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}\begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix}$$

By equality of matrices, we get

$x = 0,\ y = -5,\ z = -3.$

OR

$$AB = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$\Rightarrow AB = 6I$

$$\Rightarrow A\left(\frac{1}{6}B\right) = I$$

$$\Rightarrow A^{-1} = \frac{1}{6}(B)$$

The given system of equations is

$x - y = 3,\ 2x + 3y + 4z = 17,\ y + 2z = 7.$

The given equations can be written as

$$\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

That is $AX = D$, where $D = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$

$\Rightarrow X = A^{-1}D$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

By equality of matrices, we get

$x = 2,\ y = -1,\ z = 4$.

Q37. We have

$\vec{a}_1 = 3\hat{i} + 2\hat{j} - 4\hat{k}$, $\vec{b}_1 = \hat{i} + 2\hat{j} + 2\hat{k}$;

$\vec{a}_2 = 5\hat{i} - 2\hat{j}$, $\vec{b}_2 = 3\hat{i} + 2\hat{j} + 6\hat{k}$

$\Rightarrow \vec{a}_2 - \vec{a}_1 = 2\hat{i} - 4\hat{j} + 4\hat{k}$.

And, $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 3 & 2 & 6 \end{vmatrix} = \hat{i}(12-4) - \hat{j}(6-6) + \hat{k}(2-6)$

$\Rightarrow \vec{b}_1 \times \vec{b}_2 = 8\hat{i} - 4\hat{k}$

$\because \left(\vec{b}_1 \times \vec{b}_2\right).\left(\vec{a}_2 - \vec{a}_1\right) = 16 - 16 = 0$

As $\text{S.D.} = \dfrac{\left|\left(\vec{b}_1 \times \vec{b}_2\right).\left(\vec{a}_2 - \vec{a}_1\right)\right|}{\left|\vec{b}_1 \times \vec{b}_2\right|} = 0$

$\therefore$ The lines are intersecting and the shortest distance between the lines is 0.

Now for the point of intersection

$3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k}) = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$

$\Rightarrow 3 + \lambda = 5 + 3\mu$ ...(i)

$\Rightarrow 2 + 2\lambda = -2 + 2\mu$ ...(ii)

$\Rightarrow -4 + 2\lambda = 6\mu$ ...(iii)

Solving (i) and (ii) we get, $\mu = -2$ and $\lambda = -4$

Substituting in equation of line we get

$\vec{r} = 5\hat{i} - 2\hat{j} + (-2)(3\hat{i} + 2\hat{j} + 6\hat{k}) = -\hat{i} - 6\hat{j} - 12\hat{k}$

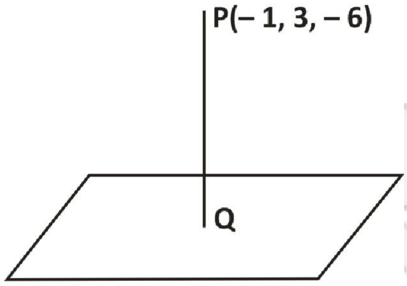
So, the point of intersection is $(-1, -6, -12)$.

OR

Let P be the given point and Q be the foot of the perpendicular.

Equation of PQ i.e., equation of $\perp^{er}$: $\dfrac{x+1}{2} = \dfrac{y-3}{1} = \dfrac{z+6}{-2}$

Let $\dfrac{x+1}{2} = \dfrac{y-3}{1} = \dfrac{z+6}{-2} = \lambda$.



Let coordinates of Q be $(2\lambda - 1, \lambda + 3, -2\lambda - 6)$

Since Q lies in the plane $2x + y - 2z + 5 = 0$

$\therefore\ 2(2\lambda - 1) + (\lambda + 3) - 2(-2\lambda - 6) + 5 = 0$

$\Rightarrow 4\lambda - 2 + \lambda + 3 + 4\lambda + 12 + 5 = 0$

$\Rightarrow 9\lambda + 18 = 0$

$\Rightarrow \lambda = -2$

$\therefore$ Coordinates of Q are $(-5, 1, -2)$.

That is, foot of perpendicular : $Q(-5, 1, -2)$.

Now Length of perpendicular $= \sqrt{(-5+1)^2 + (1-3)^2 + (-2+6)^2}$

$\therefore$ Length of perpendicular = 6 units.

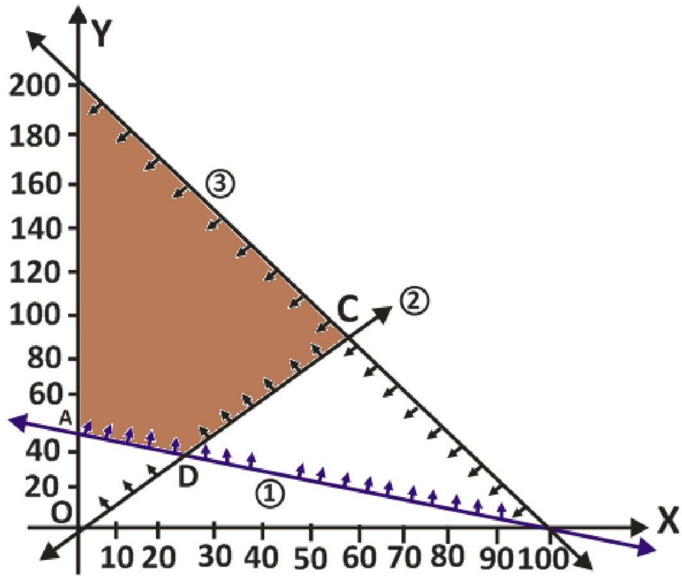
Q38. Maximize $Z = x + 2y$

Subject to $x + 2y \geq 100$ ...(i)

$2x - y \leq 0$ ...(ii)

$2x + y \leq 200$ ...(iii)

$x \geq 0, y \geq 0$.



Corner Points	$Z = x + 2y$
A(0, 50)	100
B(0, 200)	400
C(50, 100)	250
D(20, 40)	100

$\therefore$ Maximum value of $Z = 400$.

Also the maximum value is attained when $x = 0, y = 200$.

OR

(i)

Corner points	$Z = 3x - 4y$
O(0, 0)	0
A(0, 8)	–32
B(4, 10)	–28
C(6, 8)	–14
D(6, 5)	–2
E(4, 0)	12

Max. $Z = 12$ at $E(4,0)$ and, Min. $Z = -32$ at $A(0, 8)$.

(ii) Since maximum value of Z occurs at $B(4,10)$ and $C(6,8)$.

$\therefore\ 4p + 10q = 6p + 8q$

$\Rightarrow 2q = 2p$

$\Rightarrow p = q$.

So, the number of optimal solutions is infinite.